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New water density table at ITS 90

S V Gupta
National Physical Laboratory, New Delhi, India
E-mail svg@giasd101 vsnl net in

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Abstract Different expressions, for relative density of water *versus* temperature, used by various researchers since 1890 have been reviewed Recently, a good amount of work for determination of absolute or relative density of water as a function of temperature, with clear understanding of the effects of isotopic composition and air saturation on density of water, has been carried out The results have been fitted into a fairly variety of functions This has added to confusion to user scientist To circumvent this problem, a harmonized expression for relative density of water satisfying the available data, has been derived The internationally agreed value of maximum density and the temperature at which it is maximum are respectively 999.974950 kg/m^3 and 3 983035°C As the isotopic composition of water from various sources varies, the isotopic composition of standard Mean Ocean Water (SMOW) has been specified A unique expression giving the difference in density of SMOW and sample water has been stated The decrease in density due to different air saturation and temperature has been derived Water density table for SMOW using internationally agreed values of various parameters and ITS 90 has been constructed from 0 to 41°C in steps of 0 1°C

Keywords Relative density, water density, temperature variation

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1. Introduction

Basically, the following four factors govern the calculation of value of absolute density of water at any temperature :

(i) Isotopic composition of water,

(ii) Level of air saturation in water,

(iii) Temperature relationship of relative density of water,

(iv) Value of absolute density of water at the temperature of its maximum density.

Unfortunately, there is a multitude of expressions used for all the above stated factors. There is no unique expression for density correction due to air saturation. There are confusing formulae for isotopic composition of standard mean ocean water and relation for density of a water sample having the known isotopic composition. There is a number of relations for dilatation of water. There are relations derived from the experimental work, there are some relations calculated on some other basis. Similarly, the density value of the water taken by various authors at the temperature of its maximum density, varies by a few parts per million.

So in order to get universally traceable results, where density of water has been used as the standard, one, apart from knowing the actual isotopic composition of water sample and level of air saturation in water and applying the necessary corrections, should also state :

(i) The expressions used for applying ic corrections,

(ii) Density-Temperature relation used, and

(iii) The value of density used at the temperature of maximum density.

Importance of correct and uniformly adopted values of density of water at different temperatures need not be over-emphasized. For comparison of results of any volumetric measurement obtained by different laboratories, it is necessary that all laboratories use the same water density table obtained from an internationally adopted expression involving temperature and water density and other relevant parameters like maximum density and isotopic composition of water.

*Corresponding Author

2. Density of water at the temperature of its maximum density

In June 1799, while defining the kilogram de Archives as the mass of water of one decimeter cube of water, the density of water was taken as 1000 kg/m^3. Later in 1889, the kilogram—the unit of mass was declared equal to the mass of platinum iridium cylinder having the mass closest to that of kilogram de Archives and declared it as the prototype of the international kilogram. This means that the actual mass of the standard remained unchanged. But when in 1901, the litre the unit of volume was defined as the volume occupied by one kilogram of water by implication, the density of water at the temperature of its maximum density was declared as 999.972 kg/m^3. At that time, concept of isotopic composition of water was not clear and therefore was not raised. This value continued to be used till recently. Menace and Girard in 1973 [1], considering the most likely isotopic composition of water (as faced in 1901), suggested that density of water at the temperature of its maximum density should be taken as 999.975 kg/m^3. In 1994, Patterson and Morris of NML, Australia [2] measured the density of water at the temperature of maximum density as 999.97358 ± 0.00090 kg/m^3. Masui *et al* [3] measured carefully the density of water at 16°C and calculated the value of density at the temperature of maximum density as 999.9756 ± 0.0008 kg/m^3. All these density values refer to Standard Mean Ocean water known as SMOW and at the pressure of 101 325 Pa. So the problem is that even density of water at the temperature of its maximum density is known only within 2 parts per million. From the latest correspondence with BIPM, the author understands that the value *999.974950 ± 0.00084 kg/m^3 is going to be accepted for the maximum density of (SMOW). The temperature at which the density of water becomes maximum has been taken as 3.983035 ± 0.00067°C. The uncertainty quoted is 2 times standard deviation.*

3. Standard mean ocean water SMOW

Standard mean ocean water has following internationally accepted isotopic composition [4] :

$$R_{18} = (2005.2 \pm 0.45)10^{-6},$$
$$R_D = (155.76 \pm 0.05)10^{-6}, \quad (1)$$

where R_{18} in given volume of water, is the ratio of water molecules having ^{18}O to that of water molecules containing ^{16}O. Similarly, R_D is the ratio of water molecules having deuterium instead of hydrogen to the number of molecules of normal water.

Thus,

$$R_{18} = n(^{18}O)/n(^{16}O), \text{ and } R_D = n(D)/n(H).$$

The internationally accepted expression [5] giving the difference in density of SMOW and sample water is

$$\left(\rho - \rho_{(V\text{-}SMOW)}\right) = 0.233 \cdot 10^{-3} \cdot \delta^{18}O + 0.0166 \cdot 10^{-3} \cdot \delta D, \quad (2)$$

where $\delta D = [R_D(\text{sample})/R_D(\text{SMOW}) - 1]10^3$,

$$\delta^{18}O = [R_{18}(\text{sample})/R_{18}(\text{SMOW}) - 1]10^3.$$

It may be noted that different values of R_{18} and R_D and coefficients of $\delta^{18}O$ and δD in eq. (2) have been reported in the literature [6–8].

4. Effect of dissolved air on density of water

Marek [9] was perhaps the first to investigate the effect of dissolved air on density of water. He measured the difference in density of air-free and air-saturated water between 0°C to 14°C and gave results which could best be represented by a parabola having its maximum at about 5°C. That is decrease in density is maximal at 5°C. Chappuis [10] measured the difference in density of air saturated and air-free water from 5°C to 8°C. The difference in density was close to that of Marek but it remained practically the constant in the range of temperature, which Chappuis studied.

However, the states of fully saturated or completely air-free water are not realisable. So Girard and Coarasa [11] measured the difference in density of air-free water and that of water having known amount of atmospheric gases at temperatures 4, 10, 16 and 22°C and arrived at the following conclusions :

(i) The decrease in density is linearly related with the amount of air dissolved.

(ii) The decrease in density between air-free and fully-saturated water linearly decreases with temperature and becomes almost zero around 33°C and is about 0.0048 kg/m^3 at 0°C.

The amount of atmospheric gases dissolved in water are measured on the assumption that level of saturation of oxygen and nitrogen in the temperature-range under consideration, are equal [12] and level of saturation of oxygen alone represent the level of saturation of water. For calculation of level of oxygen, Winkler titration method as improved by Carpenter [13] may be used.

Bignell [14] measured the density difference between air-free and fully-saturated water at 80 points between 4°C to 20°C and gave the expression using least square method. Decrease in density $\Delta\rho$ with respect to temperature t is given by

$$\Delta\rho = -0.004612 + 0.000106t. \quad (3)$$

Uncertainty in obtaining $\Delta\rho$ from the above expression at 2σ level is

$$2.56\left[3.1\times10^{-4}+1.066\times10^{-5}(t-11.52)^2\right]^{1/2}, \quad (4)$$

t is temperature in °C on IPTS 68. The expression will remain practically same even if the temperature is changed to ITS 90.

Combining the linearity of density differences with respect to level of saturation χ and above equation, we may write density difference $\Delta\rho$ with temperature t and saturation level χ as follows :

$$\Delta\rho = (-0.004612 + 0.000106\ t)\chi \quad (5)$$

5. Earlier work on dilatation of water

A summary of the earlier work done and expressions for dilatation of water is tabulated below :

1. Chappuis [15]

$(0.9998681/d - 1)\times10^6$

$= -67.464645\,t + 8.934223\,t^2 - 0.07891946\,t^3$ (for 0°C to 10.3°C),

$= -54.7835 - 55.242760\,t + 7.945055\,t^2 - 0.04800150\,t^3$ (for 10.3°C to 13°C),

$= -114.5565 - 42.940141\,t + 7.106115\,t^2 - 0.02905759\,t^3$ (for 13°C to 41°C),

0.9998681 is relative density of water at 0°C.

2. Thiesen *et al* [16]

$$(1-d)10^3 = \left\{(t-A)^2\cdot(t+C)\right\}/\left\{B\cdot(t+D)\right\},$$

where A = 3.98°C, B = 503.570°C², C = 283°C and D = 67.26°C.

3. Steckel Szapiro [17]

Same formula as above with

A = 3.986°C, B = 407.507°C², C = 196.577°C and D = 57.002°C.

6. Tilton and Taylor [18]

Same formula with

A = 3.9863°C, B = 508.9292°C², C = 288.9414°C and D = 68.12963°C.

7. Mendeleev [19]

$$(1-d) = (t-4)^2/\left[118932+13666.75t-4.13t^2\right].$$

6. Recent work done on dilatation of water

6.1. Work done at NML, Australia :

Bell and Clarke [20] determined density of water at seven temperatures namely 3.98, 10.4, 15.4, 20.0, 30.0, 34.8, 40.1°C and expressed results as a ratio of density at temperature t and density at 3.98°C. The data was fitted in Thiesen's type expression as given below :

$$(1-d)10^3 = (t-3.98)^2\cdot(t+299.666)/\left\{519.985(t+69.223)\right\}$$

for water in equilibrium with ambient air.

The expression for air-free water is given by

$$(1-d)10^3 = (t-3.98)^2\cdot(t+238.263)/\left\{503.375(t+67.335)\right\}. \quad (6)$$

Fit was found to be excellent, though most of the values were within ± 0.3 ppm, none of the calculated and observed values was outside ± 0.8 parts per million. But the expression suffers with a fewer degrees of freedom.

Bell and Patterson [21] of same laboratory took more measurements and reported in 1984 a modified formula of Thiesen's type as

$$(1-d)10^3 = (t-3.989)^2\ (t+335.1596)/\left\{558.7666(t+72.2553)\right\}. \quad (7)$$

While Patterson and Morris [2] in 1994 determined the relative density between 0°C to 40°C and expressed their results as

$$(1-d)10^3 = (t-3.9818)^2\cdot(t+316.33808)/\left\{538.211(t+70.69973)\right\}. \quad (8)$$

The same data was also fitted to a 5-th degree polynomial in terms of $(t - t_0)$

$$\rho/\rho_{max} = 1-\left\{A(t-t_0)+B(t-t_0)^2+C(t-t_0)^3+D(t-t_0)^4+E(t-t_0)^5\right\}, \quad (9)$$

where t_0, A, B, C, D and E are

t_0 = 3.9818°C,

$A = 7.0134\times10^{-8}(°C)^{-1}$,

$B = 7.926504\times10^{-6}(°C)^{-2}$,

$C = -7.575677\times10^{-8}(°C)^{-3}$,

$D = 7.314894\times10^{-10}(°C)^{-4}$,

$E = -3.596458\times10^{-12}(°C)^{-5}$.

Here, it may be emphasised that all the results are of the same laboratory and varying too much in regards to values of various constants.

6.2. Work done at NRLM, Australia :

In 1990, Takenaka and Masui [22] measured thermal expansion of pure water having natural isotopic abundance by dilatometer method in a temperature range of 0°C to 85°C. Four sets of data were recorded comprising 72

independent ordered pairs of relative density ρ/ρ_{max} and temperature. The data was fitted to obtain the Thiesen type expression :

$$(1-d)10^3 = (t-3.98152)^2\cdot(t+396.18534)\times(t+32.28853)/\{609.6286(t+83.12333)\times(t+30.244455)\}. \quad (10)$$

It may be noticed that a factor $(t+32.2853)/(t+30.24455)$ is extra. The above expression is based on the expression, which Thiesen used in his later work [23]. He also fitted the same data in a Kell type relation [24]

Kell type expression is

$$\rho/\rho_{max} = \left(A_0 + \Sigma A_i t^{2i-1}\right)/\left(1+\Sigma B_j t^{2j}\right). \quad (11)$$

Here, i and j takes values from 1 to 4.

The values of constants are

$A_0 = 9.9986784\times10^{-1}$, $A_1 = 6.7826308\times10^{-5}$,

$A_2 = 1.0365704\times10^{-7}$, $A_3 = 1.7485485\times10^{-11}$,

$A_4 = 8.4152542\times10^{-16}$;

$B_1 = 9.0887089\times10^{-6}$, $B_2 = 1.4974442\times10^{-9}$,

$B_3 = 1.6006519\times10^{-13}$, $B_4 = 2.8106977\times10^{-18}$.

Further, he fitted the data to a 8-th degree polynomial of the form

$$\rho/\rho_{max} = \sum A_i t^{i-1}. \quad (12)$$

The coefficients obtained were

$A_1 = 9.9986785\times10^{-1}$, $A_2 = 6.7819907\times10^{-5}$,

$A_3 = -9.0858952\times10^{-6}$, $A_4 = 1.0288239\times10^{-7}$,

$A_5 = -1.4077910\times10^{-9}$, $A_6 = 1.6355966\times10^{-11}$,

$A_7 = -1.3688193\times10^{-13}$, $A_8 = 6.9699179\times10^{-16}$,

$A_9 = -1.5914816\times10^{-18}$.

The standard deviation of fitting was 0.2 parts per million in each case.

In 1991, Watanabe [25] carried out work measurement of thermal expansion of water in a temperature range of 0°C to 44°C and expressed their results as a polynomial of 6-th degree as follows :

$$\rho/\rho_{max} = \sum A_i t^{i-1}. \quad (13)$$

$A_1 = 9.9986775\times10^{-1}$, $A_2 = 6.78668754\times10^{-5}$,

$A_3 = -9.09099173\times10^{-6}$, $A_4 = 1.02598151\times10^{-7}$,

$A_5 = -1.35029042\times10^{-9}$, $A_6 = 1.32674392\times10^{-11}$,

$A_7 = -6.461418\times10^{-14}$.

So we can see the complexity due to large number of expressions, which add to our confusion.

Fortunately, the values of ρ/ρ_{max} obtained from eqs. (8 to 13) given by Patterson and Morris [2], Takenaka and Masui [22] and by Watanabe [25] agrees very well, say within 0.2 parts per million.

7. Harmonized formula for relative density of water by the author

So to harmonize the various formulae, the author obtained the values of ρ/ρ_{max} from six equations for temperatures from 0°C to 41°C in step of 1°C. Three of them namely (7), (8) and (10) were Thiesen type and other three (9), (12) and (13) were polynomials. This way, six sets of values of relative density of water were obtained. Equal weights to each data were assigned and the mean of six values for every degree from 0°C to 40°C was taken. A polynomial function for ρ/ρ_{max}, in terms of (T-3.983035) was fitted to the mean values so obtained by using least square method. 3.983035 is the internationally accepted value for the temperature on ITS 90 at which density of water becomes maximum. The fit is so good that the square root of the average sum of residual errors is only 0.1 ppm. No calculated value differs from the corresponding mean value by more than 0.17 ppm

The aforesaid range of temperature is chosen, keeping in view in which most of the metrologists are interested

The expression obtained by the author is given as

$$(1-\rho/\rho_{max})\cdot10^6 = A_1\cdot(T-3.983035) + A_2\cdot(T-3.983035)^2 + A_3\cdot(T-3.983035)^3 + A_4\cdot(T-3.983035)^4 + A_5\cdot(T-3.983035)^5, \quad (14)$$

where $A_1 = -4.971411\times10^{-2}$, $A_2 = 7.967602803$,

$A_3 = -7.960232\times10^{-2}$, $A_4 = 8.681132\times10^{-4}$,

$A_5 = -5.234077\times10^{-6}$. (15)

The value of ρ_{max} has been taken as 999.974950 kg/m^3.

It may be noticed that second term on right hand side contributes most, so its coefficient has been determined up to higher number of decimal point.

7.1. Table for density of SMOW with respect of temperature :

The density values of SMOW from 0°C to 41°C in steps of 0.1°C have been given in the Table 1. This supersedes the water density table given by Frank and Harris [26]. The values given in [26] are consistently 4 ppm lower than the values obtained by the author.

The values refer to air-free, SMOW or V-SMOW with $R_D = (155.76\pm0.05)\times10^{-6}$, $R_{18} = (2005.2\pm0.05)\times10^{-6}$ in

Table 1. Density of SMOW (Standard Mean Ocean Water) in kg/m³ on ITS 90

	Temp.	0 0	0 1	0.2	0 3	0 4	0 5	0 6	0 7	0 8	0.9
)	999	0.8431	0 8498	0.8562	0 8626	0 8687	0 8746	0 8804	0 8860	0 8914	0 8967
	999	0.9017	0 9066	0 9113	0 9159	0 9203	0 9245	0 9285	0 9323	0 9360	0 9395
)	999	0 9429	0 9461	0.9491	0 9519	0 9546	0 9571	0 9594	0 9616	0 9636	0 9654
;	999	0.9671	0 9686	0.9700	0 9712	0 9722	0 9731	0 9738	0 9743	0 9747	0.9749
(	999	0.9749	0 9748	0 9746	0 9742	0.9736	0 9729	0 9720	0 9709	0 9697	0 9684
)	999	0 9668	0.9652	0 9634	0 9614	0 9592	0 9570	0 9545	0 9519	0 9492	0.9463
,	999	0 9433	0 9401	0 9367	0 9333	0 9296	0 9258	0 9219	0 9178	0 9136	0 9092
7	999	0 9047	0 9000	0.8952	0 8903	0 8852	0 8799	0 8745	0 8690	0 8633	0 8575
8	999	0 8515	0 8454	0.8392	0 8328	0.8263	0 8196	0 8128	0 8058	0 7988	0 7915
)	999	0 7842	0 7767	0 7690	0 7613	0 7534	0 7453	0 7371	0.7288	0 7204	0 7118
10	999	0.7030	0 6942	0 6852	0 6761	0 6668	0 6574	0 6479	0 6382	0 6285	0 6185
11	999	0 6085	0 5983	0 5880	0.5776	0 5670	0 5563	0 5455	0 5345	0 5234	0 5122
12	999	0 5009	0 4894	0 4778	0 4661	0 4542	0 4423	0.4302	0 4180	0 4056	0 3931
13	999	0 3805	0 3678	0 3550	0 3420	0.3289	0.3157	0 3024	0 2889	0 2753	0 2616
14	999	0 2478	0.2338	0.2198	0.2056	0.1913	0.1769	0 1623	0 1477	0 1329	0 1180
15	999	0.1030	0 0878	0 0726	0.0572	0 0417	0 0261	0 0104	*0 9945	0 9786	0 9625
16	998	0 9463	0 9300	0 9136	0 8971	0.8804	0 8637	0 8468	0 8298	0 8127	0 7955
17	998	0 7782	0 7607	0 7432	0 7255	0 7078	0 6899	0 6719	0 6538	0 6356	0 6172
18	998	0 5988	0 5802	0 5616	0 5428	0 5239	0 5049	0 4858	0 4666	0 4473	0 4279
19	998	0 4084	0 3887	0.3690	0 3492	0 3292	0 3091	0 2890	0 2687	0 2483	0 2278
20	998	0 2072	0 1865	0 1657	0 1448	0 1238	0 1027	0 0815	0 0601	0 0387	0 0172
21	997	0 9955	0 9738	0 9520	0 9300	0 9080	0 8858	0 8636	0 8412	0 8187	0 7962
22	997	0 7735	0 7508	0 7279	0 7050	0 6819	0 6587	0 6355	0 6121	0 5887	0 5651
23	997	0 5414	0 5177	0.4938	0 4699	0 4458	0 4217	0 3974	0 3731	0 3486	0 3241
24	997	0.2994	0 2747	0 2499	0 2249	0 1999	0 1748	0 1496	0 1243	0 0988	0 0733
25	997	0 0477	0 0220	*0.9962	0 9704	0 9444	0 9183	0 8921	0 8659	0 8395	0 8130
26	996	0 7865	0 7599	0.7331	0 7063	0 6794	0 6524	0 6253	0 5981	0 5708	0 5434
27	996	0 5159	0 4883	0 4607	0 4329	0 4051	0 3772	0 3491	0 3210	0 2928	0 2645
28	996	0 2361	0 2077	0 1791	0 1504	0 1217	0 0929	0 0639	0 0349	0 0058	*0 9766
29	995	0 9473	0 9180	0 8885	0 8590	0 8293	0 7996	0 7698	0 7399	0 7099	0.6798
30	995	0 6497	0 6194	0 5891	0 5587	0 5282	0 4976	0 4669	0 4361	0 4053	0 3743
31	995	0 3433	0 3122	0.2810	0 2497	0 2183	0 1869	0 1553	0 1237	0 0920	0 0602
32	995	0.0283	*0 9964	0.9643	0 9322	0 9000	0 8677	0 8353	0 8029	0 7703	0 7377
33	994	0 7050	0.6722	0.6393	0 6063	0 5733	0.5402	0.5070	0 4737	0 4403	0 4068
34	994	0 3733	0 3397	0 3060	0.2722	0 2383	0 2044	0 1704	0 1363	0 1021	0 0678
35	994	0 0335	*0.9991	0 9646	0.9300	0 8953	0 8606	0 8257	0 7908	0 7559	0 7208
36	993	0.6857	0 6504	0 6151	0 5798	0 5443	0 5088	0 4732	0 4375	0 4017	0 3659
37	993	0 3299	0.2939	0.2578	0 2217	0 1855	0.1492	0 1128	0 0763	0 0398	0 0031
38	992	0 9665	0 9297	0.8928	0 8559	0.8189	0 7818	0 7447	0 7075	0 6702	0.6328
39	992	0 5954	0 5578	0 5202	0.4826	0 4448	0 4070	0 3691	0 3311	0 2931	0.2550
40	992	0 2168	0 1785	0 1402	0 1017	0 0633	0 0247	*0 9861	0 9474	0 9086	0 8697
41	991	0 8308									

Note Wherever asterisk (*) appears, the integral value to be taken will be one less than the figure written in the second column

The value of ρ_{max} taken is 999 974950 kg/m³

To take full advantage of the table, find the values of $\delta^{18}O$ and δD of the sample of water used, taking R_i (155 76 ± 0 05) × 10^{-1}

R_{18} = (2005 2 ± 0 05) × 10^{-6} for SMOW and the relation for density difference is

$\rho - \rho_{(V\text{-}SMOW)} = 0.233 \times 10^{-3} \cdot \delta^{18}O + 0.0166 \times 10^{-3}\ \delta D$ For correction due to air saturation use equation (5)

eq. (1). The following relation for density difference given in eq. (2) should be used.

$$\rho - \rho_{(V\text{-}SMOW)} = 0.233 \times 10^{-3} \cdot \delta^{18}O + 0.0166 \times 10^{-3} \cdot \delta D.$$

For calculating the effect of dissolved air eq. (5) may be used.

References

[1] M Menache and G Girard *Metrologia* **9** 62 (1973)

[2] J B Patterson and E C Morris *Metrologia* **31** 272 (1994)

[3] R Masui, K Fujii and M Takanaka *Metrologia* **32** 333 (1995/96)

[4] *Recommended Reference Materials for the Realisation of Physico Chemical Properties* (ed) K N Marsh (Oxford Blackwell Scientific Publications) p 14 (1987)

[5] M Menache, C Beaverger and G Girard *Annales de Hydrographiques 5-th Series* **Vol. 6** No 750 p 37 (1978)

[6] H Craig *Science* **133** 1833 (1961)

[7] M Menache *Metrologia* **3** 58 (1967)

[8] G Girard and M Menache *Metrologia* **7** 83 (1971)

[9] W J Marek *Ann Phys Chem* **44** 171 (1891)

[10] P Chappuis *Trav Mem Bur Int Poids Mes* **14** D1 (1910)

[11] G Girard and M J Coarasa *Effect of Dissolved Air on the Density of Water Precision Measurements and Fundamental Constants II* (eds) B N Taylor and W D Phillips p 453 (1984)

[12] *CRC Handbook of Chemistry and Physics* (ed) David R Lida 77-th edn p 6 (1998)

[13] J H Carpenter *Limnol Oceanogr* **10** 135 (1965)

[14] N Bignell *Metrologia* **19** 57 (1983)

[15] P Chappuis *Trav Mem Bur. Poids Mes.* **13** D1 (1907), *Trav Mem Bur. Poids. Mes* **14** D63 (1910)

[16] M Theisen, K Scheel and H Diesselhorst *Wiss. Abh Phys-tech Reichsanst* **3** 1 (1900)

[17] F Steckel and S Szapiro *Trans Faraday Soc* **59** 331 (1963)

[18] L W Tilton and J K Taylor *J Res Nat Bur Stand (Wash.)* **18** 205 (1937)

[19] D I Mendeleev *Vremenik Glavnoj Palati Mer I Vesov* 3-rd Partie 133 (1897)

[20] G A Bell and A L Clarke *A Determination of Dilatation of Pure Water of known Isotopic Composition Atomic Masses and Fundamental Constants* (eds) J H Sanders and A H Wapstra (New York Plenum) **Vol. 5** p 615 (1976)

[21] G A Bell and J B Patterson *Density Standards, The Density and Thermal Dilation of Water Precision Measurements and Fundamental Constants II* (eds) B N Taylor and W D Phillips p 445 (1984)

[22] M Takenaka and R Masui *Metrologia* **27** 165 (1990)

[23] M Thiesen *Phys Tech Reichs Wiss Abhandl* **4** 1 (1904)

[24] G S Kell *J Phys Chem Ref Data* **6** 1109 (1977)

[25] H Watanabe *Metrologia* **28** 33 (1991)

[26] Frank E Jones and G L Harris *J Res Natl Inst Stand Technol* **97** 335 (1992)